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ACTION OF AN EXPLOSIVE PLASTIC WAVE ON A PLATE

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The action of a transient loading on an infinitely elastic plate, freely covering the surface of an ideal compressible liquid, was discussed in [1-3]. A review of work on the dynamics of a plate under the action of a transient loading is given in [4].

The motion of a rectangular plate of finite dimensions is considered, under the action of a plastic, plane, explosive compression shock wave, incident at an angle. The plate is the side of a rectangular cavity filled with an ideal compressible liquid. The cavity is in a dense medium (earth) and is bounded by rigid immovable walls. In this same medium, at a distance of $\eta_0 a$ and at an angle α to the surface of the plate, a plane layer of an explosive charge with thickness $2a$ is detonated (Fig. 1), where $\eta_0 = (z \cos \alpha)/a$ is the dimensionless distance. The explosive charge, when detonated, is converted instantaneously into gas at high pressure without change of volume, as a result of which an initial pressure p_2 is applied to the surface of the medium AB, which causes the formation in the medium of a plastic compression shock wave. The velocity of the front and the parameters of motion of the medium are known (determined by a computational or experimental method [5, 6]).

It will be assumed that the diagram of compression of the medium is described by a power law and has an asymptote, corresponding to the pressure, which tends to infinity. Then the pressure at the front of the wave is determined by the formula [5]

$$p_1 = C_1(\eta_0 + \eta_1)^{\lambda},$$

where $C_1 = p_2 \beta A_0^{m+2}$; $\lambda = \omega(m + 2)$; η_1 is a dimensionless distance, measured in the direction normal to the front; the quantities β, A_0, m, and ω depend on the exponent of compression of the medium n and the isentropy exponent for the detonation products and are found by well-known relations [5].

Using the results of [7, 8], we write the expression for the pressure of the shock plastic wave at the surface at the instant of reflection in the form

$$p = p_1(1 + q) \cos \alpha,$$

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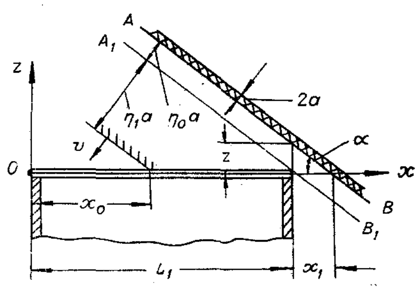

Fig. 1

where the quantity (1 + q) is the coefficient of reflection for normal incidence; q is determined from the relation $(1 + q^{-1})^n = 1 + q$. Since $n \geq 1$, the value of $q \geq 1$ and the coefficient of reflection for the shock plastic wave is always greater than 2. For sandy soils of broken structure, the values of n are 2.5 to 3.0, and, correspondingly, the quantity 1 + q = 2.84 to 3.0, which coincides well with the experimental result [9], carried out in sandy soils. The velocity of the obstacle at the instant of reflection has an effect on the coefficient of reflection. Calculations show, however, that even in the case of equality of the velocities of the obstacle and of the particles in the incident wave, the quantity q varies within limits of not more than 10% [8], so that we can neglect the effect of the velocity of the obstacle.

The velocity of the front and the stress at the front decrease in proportion with the propagation of the explosion wave. Therefore, a distributed load acts on the plate, the front of which is moving also with a monotonically decreasing velocity. The law of motion of the loading front is found from the relations [5] (see Fig. 1)

$$x_0 = L_1 - (\eta_1 a/\sin\alpha), \quad t_1 = (\rho_0/p_2)^{1/2}\,[a/A_0(1-\omega)]\,\eta_1^{1-\omega}, \tag{1}$$

where ρ_0 is the density of the medium; L_1 is the length of the plate in the direction of the x axis. The time t_1 is measured from the instant of motion of the shock front from the plane A_1B_1. The law of change of pressure after reflection will be assumed to be in the form $[1 - (t/t_0)]^s$, where $s \geq 1$; t is the time, measured from the instant of reflection; and t_0 is the time of action of the wave on the plate. The action of the plastic wave on the plate terminates at the instant when the wall of the cavity AB stops; the law of motion for this has the form

$$\bar{x} = 1 + \varepsilon_* A_0^{-1/\omega} + [\beta A_0^m/(1+m\omega)]\,(\eta_0+\eta_1)^{1+m\omega}, \quad \eta = \eta(t),$$

where $\bar{x}$ is the dimensionless displacement of the boundary of the cavity; ε_* is the limiting value of the deformation of the medium.

Thus, the function for the normal pressure on the plate at a point with a fixed coordinate is written in the form

$$p = \begin{cases} p_0\,[1+(\eta_1/\eta_0)]^{\lambda}\,[1-(t/t_0)]^s\,(1+q)\cos\alpha, & \eta_1 > 0, \\ 0, & \eta_1 = 0, \end{cases}$$

where p_0 is the pressure at the wave front at the instant $\eta_1 = 0$.

In the system of coordinates xyz (see Fig. 1), the equation describing the motion of the plate is assumed to be in the form

$$\rho_1 h(\partial^2 w/\partial t^2) + D\nabla^2\nabla^2 w + p_* = -p(x,\ t) \text{ when } t_1 > 0, \tag{2}$$

where w and h are the displacement and thickness of the plate; $\nabla^2 = \partial^2 w/\partial x^2 + \partial^2 w/\partial y^2$; D is the cylindrical rigidity; ρ_1 and ν are the density and Poisson's coefficient for the material; p_* is the pressure of the liquid; and p(x, t) is the loading function. We shall neglect the effect of tangential forces on the motion of the plate and we shall assume that there are no perturbations ahead of the incident wave front.

The motion of the plate must satisfy the conditions of immobilization and the initial conditions

$$w = \partial w/\partial t = 0,\ t_1 = 0. \tag{3}$$

When the plate comes into contact with the liquid, then as a result of the motion of the plate, a wave motion originates in the liquid — a radiation wave. Suppose that φ is the velocity potential in the liquid. The displacements w and φ must satisfy the boundary conditions

$$\partial w/\partial t = \partial\varphi/\partial z,\ z = 0,\ 0 \leqslant x \leqslant L_1. \tag{4}$$

Using the hypothesis of plane reflection [10] and condition (4), we write the expression for the pressure of the liquid:

$$p_* = \rho_2 c(\partial w/\partial t), \tag{5}$$

where ρ_2 is the density of the liquid and c is the velocity of sound.

We determine the motion of the plate under the action of a unit step-loading, the front of which is moving according to the law (1):

$$p(x) = H(x - x_0),\quad H(x - x_0) = \begin{cases} 1 & \text{when } x \geqslant x_0, \\ 0 & \text{when } x < x_0, \end{cases}$$

$$x_0 = L_1 - (\eta_1 a/\sin\ \alpha).$$

The expression for the bending of the plate is assumed to be in the form

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} f_{mn}(t) X_m(x) Y_n(y). \tag{6}$$

The functions $X_m(x)$ and $Y_n(y)$ are chosen from a number of fundamental functions, starting from the conditions of immobilization of the plate at the edges. If the edges are freely supported, then

$$X_m(x) = \sin\ \beta_m x,\ Y_n(y) = \sin\gamma_n y,\ (\beta_m = \pi m/L_1,\ \gamma_n = \pi n/L_2),$$

where L_2 is the dimension of the plate in the direction of the y axis. For fixed edges when x = 0 and L_1, we have

$$X_m(x) = \sin\lambda_m x - \operatorname{sh}\lambda_m x - g_m(\cos\ \lambda_m x - \operatorname{ch}\ \lambda_m x), \tag{7}$$

where

$$g_m = (\sin\mu_m - \operatorname{sh}\mu_m)/(\cos\mu_m - \operatorname{ch}\mu_m),\ \ \mu_m = \lambda_m L_1 = (2m + 1)\pi/2$$

is the characteristic number of the function (7) and the root of the equation $\cosh\mu_m \cos\mu_m - 1 = 0$, m = 1, 2, 3, ... A similar expression is taken also for $Y_n(y)$ if the edges are fixed when y = 0 and L_2.

We substitute expressions (5) and (6) in Eq. (2) and we solve it by the Bubnov–Galerkin method; then, multiplying it by $X_k(x)$ and $Y_j(y)$ and integrating within the limits from 0 to L_1 and from 0 to L_2, we arrive at the equation

$$\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \left[\delta_{km,jn}\left(f''_{mn} + 2\varkappa f'_{mn}\right) + \Omega^2_{km,jn} f_{mn} + \varkappa Q_{kj}/\rho_1 h L_1 L_2\right] = 0,$$

where $\delta_{km,jn}$ is the Kronecker symbol (1 when m = k, j = n; 0 when m ≠ k, n ≠ j; m = k, n ≠ j; m ≠ k, n = j); $\Omega^2_{mk,jn}$ is the square of the natural frequency of oscillation of the plate; $2\varkappa = \rho_2 c/\rho_1 h$;

$$Q_{kj} = \int_{x_0}^{L_1} \int_{0}^{L_2} X_k(x) Y_j(y)\, dx dy;$$

and the primes denote differentiation with respect to the time t_1.

For a freely supported plate we have (during integration k and j are replaced by m and n)

$$\Omega^2_{mn} = D\left(\beta^2_m + \gamma^2_n\right)^2/\rho_1 h,\quad Q_{mn} = 2\left[\cos(\pi m x_0/L_1) - (-1)^m\right],$$
$$n = 1, 3, 5, \ldots;\quad \chi = 4.$$

For a plate fastened at the edges

$$\Omega^2_{mk,jn} = D[F_{mk} + 2H_{mk}H_{nj} + F_{nj}]/\rho_1 h,$$
$$Q_{kj} = 4[2(-1)^{k+1} + \cos\lambda_k x_0 + \operatorname{ch}\lambda_k x_0 + g_m(\sin\lambda_k x_0 - \operatorname{sh}\lambda_k x_0)],$$
$$j = 1, 3, 5, \ldots; \quad \chi = 1;$$

$$F_{mk} = \begin{cases} \lambda_m^4 & \text{when } m = k, \\ 0 & \text{when } m \neq k; \end{cases} \quad H_{mk} = \begin{cases} \lambda_m^2[1-(2/\mu_m)] & \text{when } k = m, \\ \dfrac{2\lambda_m^2(\lambda_k - \lambda_m)}{\lambda_m\mu_m + \lambda_k\mu_k} & \text{when } k \neq m,\ k+m = \text{odd}, \\ \dfrac{4\lambda_m^2\lambda_k^2}{(\lambda_k^2+\lambda_m^2)(\mu_m+\mu_k)} & \text{when } k \neq m,\ k+m = \text{even}. \end{cases} \tag{8}$$

The expressions for H_{nj} and F_{nj} are obtained from Eq. (8) by substitution of the corresponding indices.

Using relation (1), we represent the expression Q_{kj} (x_0) in the form Q_{kj} (t_1). Then for a freely supported plate we have the equation

$$f''_{mn} + 2\varkappa f'_{mn} + \Omega^2_{mn} f_{mn} = \left[8(-1)^{m+1}/\pi^2 mn\rho_1 h\right](\cos\Delta t_1^{s_1} - 1), \tag{9}$$

where

$$s_1 = 1/(1-\omega); \quad \Delta = \left[(\rho_0/p_2)^{1/2} a^{\omega}(L_1 \sin\alpha)^{1-\omega}/A_0(1-\omega)\right]^{-s_1}.$$

We reduce Eq. (9), which satisfies the initial conditions (3), to the form

$$f_{mn} = \frac{8(-1)^{m+1}}{\pi^2 mn\rho_1 h(r_2 - r_1)} \int_0^{t_1} \left(e^{r_2(t_1-\theta)} - e^{r_1(t_1-\theta)}\right)(\cos\Delta\theta^{s_1} - 1)\,d\theta, \tag{10}$$

where $r_{1,2}$ are the roots of the characteristic equation.

In the case of a unit step-loading, moving with constant velocity v, so that x_0 = vt, and for real values of $r_{1,2}$ solution (9) has the form

$$f_{mn} = -\frac{8}{\pi^2 mn\rho_1 h}\left\{\frac{1}{r_2 - r_1}\left[\frac{2\varkappa\Delta_1^2}{\Delta_3}\left(e^{r_1 t} - e^{r_2 t}\right) + \left(\frac{\Delta_2}{\Delta_3} - \frac{(-1)^m}{\Omega^2_{mn}}\right)\left(r_1 e^{r_2 t} - r_2 e^{r_1 t}\right) + \frac{\Delta_2}{\Delta_3}\cos\Delta_1 t + \frac{2\varkappa\Delta_1}{\Delta_3}\sin\Delta_1 t - \frac{(-1)^m}{\Omega^2_{mn}}\right]\right\}, \tag{11}$$

where $\Delta_1 = m\pi v/L_1$; $\Delta_2 = \Omega^2_{mn} - \Delta_1$; $\Delta_3 = \Delta_2^2 + (2\varkappa\Delta_1)^2$. Putting $\varkappa$ = 0 and x_0 = vt in Eq. (9), we find the solution without taking into account the effect of the liquid:

$$f_{mn} = -\frac{8}{\pi^2 mn\rho_1 h}\left[\left(\frac{(-1)^m}{\Omega^2_{mn}} - \frac{1}{\Delta_2}\right)\cos\Omega_{mn} t + \frac{1}{\Delta_2}\cos\Delta_1 t - \frac{(-1)^m}{\Omega^2_{mn}}\right]. \tag{12}$$

We shall call the values of the velocity of the wave front, satisfying the condition $v_k = \Omega_{mn} L_1/\pi m$, the critical values. It can be seen from Eqs. (11) and (12), with the critical values of the velocity of motion of the loading front $v = v_k$, that the deflections of the plate have higher values and depend on the magnitude of the hydrodynamic deformation. It follows from Eq. (12) that in the absence of the deforming effect of the liquid ($\varkappa$ = 0) and $v = v_k$, the deflections of the plate tend to infinity (similar to the phenomenon of resonance in oscillating systems).

The solutions of Eqs. (10)-(12) can be considered like an effect function, and w can be determined by the action of a loading of arbitrary shape by means of the integral

$$f^*_{mn} = p(x_0) f_{mn}(x_0) + \int_{x_0}^{L_1} f_{mn}(x_0 - x)[\partial p(x)/\partial x]\,dx,$$

where $p(x_0)$ is the loading value at the front and p(x) is a function, characterizing the change of loading behind the front.

Numerical Example. The plate has dimensions of L_1 = 200 cm, L_2 = 100 cm, h = 1 cm, E = $2 \cdot 10^6$ kg/cm², ν = 0.3, $\rho_1 = 8 \cdot 10^{-6}$ kg·sec²/cm⁴. Water is contained in the cavity for which $\rho_2 = 1.02 \cdot 10^{-6}$ kg·sec²/cm⁴, and

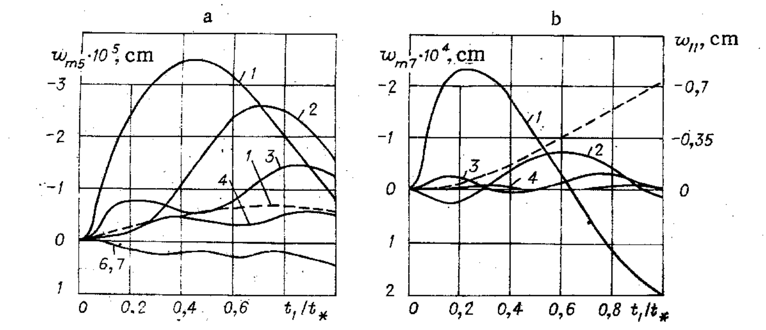

Fig. 2

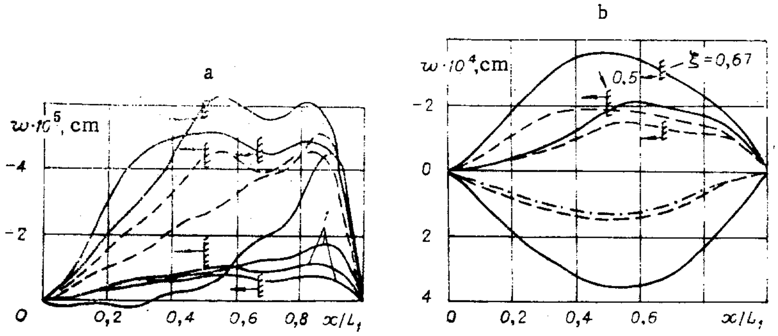

Fig. 3

$c = 1.5 \cdot 10^5$ cm/sec. The deflections w are determined for a unit step-loading, moving with a constant velocity $v = 6 \cdot 10^4$, $8.76 \cdot 10^4$, and $9 \cdot 10^4$ cm/sec. The results of the calculations are shown in Figs. 2 and 3.

Figure 2 shows the changes of deflection w_{mn} in a given section of the plate $x/L_1 = 0.8$ as a function of the dimensionless time. Here t_* is the time of advance of the loading front from $x = L_1$ to $x = 0$. The dashed and solid lines in Fig. 2a correspond to the values n = 3 and 5, the solid lines in Fig. 2b correspond to the value n = 7, and the dashed curve shows the change of deflection w_{11} in the section of the plate $x/L_1 = 0.5$, without taking account of the effect of the liquid. The scale for this curve is shown on the right. When determining w without taking into account the effect of the liquid, we can limit ourselves to one term of the series m = 1 and n = 1 with a high degree of accuracy (the contribution of the other terms of the series is not more than 2%). Curves 1-4, 6, and 7 in Fig. 2 correspond to the figures m = 1-4, 6, and 7.

The curves in Fig. 3 correspond to deflections w along the length of the plate for certain fixed positions of the loading front $\xi = x_0/L_1 = 0.67$; 0.5; and 0. The position of the front is shown by a dashed line. It was assumed in the calculation that in the direction of the y axis at a length L_2, a specified number n of half-waves is formed, and summation of the series over m was carried out from m = 1 to m = 8. The curves 1 in Fig. 3a correspond to the value n = 3, and the others to the value n = 5; in Fig. 3b the value n = 7 is taken. The solid lines relate to the velocity $v = 6 \cdot 10^4$ cm/sec and the dashed and dashed-dot lines relate to the velocities $v = 8.76 \cdot 10^4$ and $9 \cdot 10^4$ cm/sec.

It can be seen from Fig. 2b (dashed line) that in the absence of liquid in the cavity the deflection of the plate during the time t_* increases monotonically and reaches a magnitude of 0.7 h.

Even if the plate comes into contact with the liquid, the motion of the plate is close to aperiodic and the magnitude of the deflection is much less than the deflection without taking into account the effect of the liquid, which is due to the large damping by the liquid. The change of w_{mn} with time for different modes of m and n is not identical. The deflections when m = 1 and n = 5 and 7 are generated considerably more rapidly. Therefore, the appearance can be expected here of deflections with a single half-wave in the direction of the x axis and several half-waves in the direction of the y axis (m = 1, n > 3).

It follows from comparison of the curves 1 in Fig. 2a that the deflection w_{13} is considerably less than the magnitude of w_{15}. The deflection of a plate with the number of half-waves m = 1 and n = 1 and 3 must be accompanied by large changes in volume of the liquid. Because of the weak compressibility of the liquid, these deflections are small. With increase of the number n, the liquid can overflow from one region to another and the magnitude of w_{mn} increases somewhat.

Analysis of the data plotted in Figs. 2 and 3 shows that the magnitude and nature of change of the deflection of the plate depends on the shape of the wave formation, the position of the loading front, and its velocity. With increase of velocity v, the deflections decrease.

For $t_0 > t_*$, assuming $Q_{mn} = 4$, $m = 1, 3, 5, \ldots$, $n = 1, 3, 5, \ldots$, $Q_{kj} = 16$, $k = 1, 3, 5, \ldots$ and $j = 1, 3, 5, \ldots$, and using the values obtained for w_{mn} and w'_{mn} as the starting values, the further change of deflection can be determined.

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FRACTURE OF CYLINDRICAL SHELLS BY THE ACTION OF PERIODIC SHOCK WAVES

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It is well-known that in a closed tube, to one end of which is applied a sinusoidal piston movement, nonlinear longitudinal oscillations originate, which in the vicinity of the natural frequencies transform to periodic shock waves [1-13]. Similar oscillations originate during the unstable operation of the combustion chamber of engines [14-16]. In the experiments carried out up to now, the amplitude achieved 0.36 bar with an average pressure in the tube of 1 bar [4, 8, 13]. Forced axisymmetrical oscillations of thin-walled shells under the action of periodic shock waves inside their cavity have been studied in [17]. A relatively good carrying capacity is characteristic of them. This is explained by the fact that the oscillations are accompanied by a predominantly stretching–compression of the cross-section of the shell. Moreover, experiments were carried out at frequencies close to the natural frequencies of the gas column $\omega_k = k\pi a/L$ (in order to produce shock waves in the gas) and axisymmetrical oscillations of the shell $\Omega_i \approx \Omega_0 \approx c/R$ remote from the natural frequencies. Here a and c are the propagation velocities of sound in the gas and in the shell; L and R are the total length of the tube and the radius of the middle surface of the shell.

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